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Dynamic Behavior of a Beam Drag-Force Anemometer

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Dynamic Behavior of a Beam Drag-Force Anemometer

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SUMMARY

This report describes an experiment designed to determine the dynamic behavior of a drag-force anemometer in high-frequency, unsteady flow. In steady flow the output of the anemometer is proportional to stream velocity head and flow angle. Fluid mechanics suggests that, in unsteady flow, the output would also be proportional to the rate of change of fluid velocity. It was determined from the experiment described herein that effects due to the rate of change of fluid velocity are negligible for the probe geometry and frequencies involved.

INTRODUCTION

The drag-force anemometer considered in this report is a cantilevered beam with attached strain gages. It is capable of measuring both static and dynamic velocity head and flow angle.

In contrast to the hot-wire anemometer, which is an energy-balance device, the drag-force anemometer has an output that is proportional to the force exerted on the beam by the moving fluid. For steady-state conditions the force is just that due to the velocity head of the fluid. However, fluid mechanics suggests that, if the velocity is not constant, an additional term is present that is proportional to the rate of change of fluid velocity.

If this term were not negligible, there would be a spurious response at high frequencies since the velocity derivative would be higher there. It would also be more difficult to extend the frequency response of the beam electronically (ref. 1).

This report describes the effect of a velocity derivative term and the experiments performed to detect that effect.

SYMBOLS

A_D area of drag-force anemometer, m^2

C velocity derivative coefficient, $mV/(m/sec)^2$

C_D drag coefficient

D determinant

D_D drag, N

f	frequency, Hz
f_n	natural frequency of drag-force anemometer, Hz
H	$10^{(h/10)}$
h	spacing by which equal-error curve misses specified data points
K	offset parameter in curve-fitting scheme
R	output amplitude of drag-force anemometer
r_i	value of i^{th} data point
s_i	$10^{(r_i/10)}$
U	velocity of fluid, m/sec
U_0	steady part of flow, m/sec
x	electrical output of drag-force anemometer, mV
x_0	dc part of signal, mV
z	$(f/f_n)^2$
α	curve-fitting parameter, $1 - 2\zeta^2$
β	velocity derivative parameter, $(C/2\kappa)^2$
β'	$\gamma\beta$
γ	$10^{(K/10)}$
ζ	damping coefficient of drag-force anemometer
κ	velocity head coefficient, $mV/(m/sec)^2$
λ	turbulence amplitude of moving fluid
ρ	density, kg/m^3
φ	phase shift in drag-force anemometer output
ω	$2\pi f$, sec^{-1}
ω_n	$2\pi f_n$, sec^{-1}

THEORY

The drag-force anemometer considered in this report is a cantilevered beam with strain gages attached near the fixed end (fig. 1). It is a second-order system (ref. 2) and is described by the second-order differential equation

$$\frac{\ddot{x}}{\omega_n^2} + \frac{2\zeta}{\omega_n}\dot{x} + x = D_D(t) \tag{1}$$

The right member of equation (1), $D_D(t)$, denotes the drag on the beam caused by the moving fluid. The natural frequency $f_n = \omega_n/2\pi$ is a function of the length, thickness, and composition of the beam; the damping coefficient ζ is a function of the composition only; $x(t)$ is the output signal.

For steady flow the drag is proportional to the exposed area of the beam A_D and to the velocity head $1/2\rho U^2$. That is, for a steady flow of velocity U_0

$$D_D(t) = C_D A_D \left(1/2\rho U_0^2\right) \tag{2}$$

The constant of proportionality C_D is called the drag coefficient.

The theory of fluid mechanics suggests that for unsteady flow, where U is not constant, another term is present that is proportional to the rate of change of the fluid velocity (ref. 3). For this case equation (1) is rewritten as

$$\frac{\ddot{x}}{\omega_n^2} + \frac{2\zeta}{\omega_n}\dot{x} + x = \kappa U^2(t) + \frac{CU_0}{\omega_n}\dot{U}(t) \tag{3}$$

Written in this way, both κ and C have the same units, $mV/(m/sec)^2$. The case where $C = 0$ corresponds to the absence of the velocity derivative term.

The theoretical response of the probe can be examined by assuming that the flow velocity $U(t)$ is given by

$$U(t) = U_0(1 + \lambda \sin \omega t) \qquad \text{for } \lambda << 1 \tag{4}$$

which is a steady flow of velocity U_0 plus a small oscillatory turbulence of magnitude λU_0 and frequency $\omega/2\pi$.

With $U(t)$ given by equation (4)

$$U^2(t) = U_0^2(1 + 2\lambda \sin \omega t + \lambda^2 \sin^2 \omega t)$$

$$\simeq U_0^2(1 + 2\lambda \sin \omega t)$$

to first order in λ, and $dU/dt = \lambda U_0 \omega \cos \omega t$. The expression for the beam output (eq. (3)) becomes

$$\frac{\ddot{x}}{\omega_n^2} + \frac{2\zeta}{\omega_n}\dot{x} + x = \kappa U_0^2 + 2\lambda\kappa U_0^2 \sin \omega t + \lambda C U_0^2 \frac{\omega}{\omega_n} \cos \omega t \tag{5}$$

The solution of equation (5) can be written in the form

$$x = \kappa U_0^2 \left[1 + 2\lambda R \sin(\omega t - \varphi)\right] \tag{6}$$

where

$$R = \left[\frac{1 + \beta\frac{\omega^2}{\omega_n^2}}{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + 4\zeta^2\frac{\omega^2}{\omega_n^2}}\right]^{1/2} \tag{7}$$

is the amplitude of the output as a function of frequency and

$$\varphi = \tan^{-1} \frac{\frac{\omega}{\omega_n}\left[2\zeta - \sqrt{\beta}\left(1 - \frac{\omega^2}{\omega_n^2}\right)\right]}{1 - \frac{\omega^2}{\omega_n^2} + 2\sqrt{\beta}\,\zeta\frac{\omega^2}{\omega_n^2}} \tag{8}$$

is the phase shift. The dimensionless parameter β, which appears in equations (7) and (8), is defined as

$$\beta = \left(\frac{C}{2\kappa}\right)^2 \tag{9}$$

It indicates the relative strength of the velocity derivative effect.

As can be seen from equation (7), the output of the beam depends on the frequency as well as on the parameters β and ζ. Response curves for various values of β are shown in figure 2 for the highly underdamped case where $\zeta = 0.01$. The quantity plotted is

$$\text{Relative output (dB)} = 20 \log \frac{x(f)}{x(0)}$$

$$= 10 \log \frac{1 + \beta\left(\frac{f}{f_n}\right)^2}{\left[1 - \left(\frac{f}{f_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{f}{f_n}\right)^2} \tag{10}$$

EXPERIMENTAL SETUP

One of the drag-force anemometers that was tested is shown in figure 1. The probe consists of a commercially available silicon beam 0.25 millimeter thick with an unsupported length of 2.5 millimeters, the outer 1.5 millimeters of which is exposed to the flow. A four-arm, diffusion-bonded strain-gage bridge is on one side of the beam at the base. Because silicon is brittle like glass, the anemometer is fragile and care is required in handling it. The natural frequency of the beam in the first bending mode is about 40 kilohertz. Details of the anemometer are given in reference 2.

Flow excitation to the anemometer was supplied by a 9.5-millimeter-radius free jet that was operated at a Mach number of 0.3. The jet turbulence intensity was about 2 percent. The silicon beam was mounted 5 millimeters from the end of the free-jet nozzle. To determine the true frequency spectrum of the free jet, a hot-wire anemometer with a frequency response of at least 80 kilohertz was used. The hot wire was placed at the same location in the free jet as the beam, with the wire parallel to the plane of the jet exit.

The output of the anemometer was fed into a spectrum analyzer that had a bandwidth of 20 hertz to 50 kilohertz. The analog output of the analyzer was connected to the plotter to produce hard copies of anemometer output as a function of frequency.

RESULTS

Figure 3 shows the output signal spectrum from the drag-force anemometer, and figure 4 shows the spectrum of the open jet obtained with the hot-wire anemometer.

Since the hot-wire anemometer has a frequency response of at least 80 kilohertz, it is assumed that figure 4 is also the true spectrum of the jet. Since the true spectrum of the jet is not flat with frequency, an appropriate correction must be made to the output signal spectrum of figure 3 in order to obtain the desired frequency response of the drag-force anemometer. The amount of the correction at each frequency is equal to the number of decibels that must be added to the jet spectrum (fig. 4) in order to bring the curve up to zero. Figure 5 shows the corrected spectrum for the drag-force anemometer, which is identical to its frequency response curve.

The theoretical response equation (eq. (10)) was then fitted to the curve of figure 5 by using the "min-max" method in order to determine the parameters β and ζ. The details of this procedure are given in the appendix. In using this procedure the natural frequency f_n is considered to be known since for a highly underdamped system the peak occurs at nearly f_n (ref. 2).

The experimental and fitted curves are shown in figure 6. The maximum deviation h between the experimental and analytical curves was 0.9 decibel. The parameter h is the figure of merit for the min-max method.

The fitted curve corresponded to ζ and β values of 0.03 and 0.2, respectively. A second drag-force anemometer was tested and gave values for ζ and β of 0.02 and -0.3, respectively. The figure of merit h for this second curve fit was 1.9 decibels. Of course, β cannot really be negative since it is a squared quantity, but the indication is that $\beta \ll 1$ for both anemometers tested. Therefore the velocity derivative effect can be ignored, at least up to $f/f_n = 1.25$, or to about 50 kilohertz for the high-frequency silicon beams considered herein ($f_n \simeq 42$ kHz).

CONCLUDING REMARKS

The results of theory and experiment on miniature, beam drag-force anemometers show that the velocity derivative term in the drag-force function can be neglected for a wide range of frequencies. For the case shown, where silicon beams with a natural frequency f_n of 42 kilohertz were used, this velocity derivative term could be neglected to at least 50 kilohertz. This range should certainly be adequate since in most cases the user is interested only in frequencies less than about $0.25\ f_n$.

Electronically extending the frequency range of the anemometer should present no problem.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, February 20, 1980,
505-04.

APPENDIX - CURVE FITTING

The curve to be fitted has the form

$$R = 10 \log \frac{1 + \beta\left(f/f_n\right)^2}{\left[\left(f/f_n\right)^2 - 1\right]^2 + 4\zeta^2\left(f/f_n\right)^2} + K \qquad \text{(A1)}$$

The argument of the logarithm is from equation (7); the logarithm occurs because the spectrum analyzer output is in decibels. The extra term K is added because the zero point is arbitrary.

The usual method of curve fitting is least squares, where the fitted curve satisfies the criterion that the sum of the squares of the deviations between the analytical and experimental curve is a minimum. Because of the nonlinearity of equation (A1), it was necessary to iterate to find β and ζ by using the least-squares method. However, the iteration diverged. Furthermore it tended to produce a good fit only near the peak at $f/f_n = 1$.

For these reasons the least-squares approach was discarded. Instead a curve-fitting technique called the min-max method (ref. 4) was used. In this method the curve fitted to the data satisfies the criterion that the maximum error between the two curves be a minimum. The existence and uniqueness of the min-max curve are discussed in the reference, as are the equal-error property and the exchange algorithm.

The equal-error property is that property of a min-max curve by which it deviates from the data with alternating sign by an amount h, the maximum error, exactly $n + 1$ times, where n is the number of parameters that describe the curve. The exchange algorithm makes use of this property in actually fitting the curve to the data. The procedure starts with the fitting of an equal-error curve to $n + 1$ arbitrarily selected data points. If no other data point has an error larger than these $n + 1$ points, the desired curve has been found. If one or more other points do have larger errors, the point having the largest error is used to replace one of the original points in such a way that the errors are still of opposite sign. A new equal-error curve is then fitted to this new set of $n + 1$ points, and the process continues until no data point has an error greater than the $n + 1$ points used in constructing the curve.

In this case, the function to be used to fit the curve of figure 5 is given by equation (A1), so that there are three parameters: ζ, β, and K. At this point it is convenient to make the change of variables

$$\left.\begin{aligned} z &= \left(\frac{f}{f_n}\right)^2 \\ \alpha &= 1 - 2\zeta^2 \end{aligned}\right\} \tag{A2}$$

In terms of these new variables, equation (A1) becomes

$$R(z) = 10 \log\left(\frac{1 + \beta z}{z^2 - 2\alpha z + 1}\right) + K \tag{A3}$$

Now let z_i be the locations of four arbitrary data points, and r_i their magnitudes, for $i = 1, 2, 3, 4$. The equal-error curve is to miss each of these points by an amount h, with alternating sign. That is, $R(z_i) - r_i = \pm h$, so that there are four equations in the four unknowns α, β, K, and h. These equations are

$$\left.\begin{aligned} 10 \log\left(\frac{1 + \beta z_1}{1 - 2\alpha z_1 + z_1^2}\right) + K - r_1 &= h \\ 10 \log\left(\frac{1 + \beta z_2}{1 - 2\alpha z_2 + z_2^2}\right) + K - r_2 &= -h \\ 10 \log\left(\frac{1 + \beta z_3}{1 - 2\alpha z_3 + z_3^2}\right) + K - r_3 &= h \\ 10 \log\left(\frac{1 + \beta z_4}{1 - 2\alpha z_4 + z_4^2}\right) + K - r_4 &= -h \end{aligned}\right\} \tag{A4}$$

These equations can be simplified by dividing through by 10 and then taking the antilog of each equation. For instance, the first equation becomes

$$10\,\frac{1+\beta z_1}{1-2\alpha z_1+z_1^2}\,10^{(K/10)} = 10^{(r_1/10)}\,10^{(h/10)}$$

From the definitions

$$\left.\begin{aligned}\gamma &= 10^{(K/10)}\\ s_i &= 10^{(r_i/10)}\\ H &= 10^{(h/10)}\end{aligned}\right\} \qquad \text{(A5)}$$

this equation becomes

$$\frac{\gamma+\beta' z_1}{1-2\alpha z_1+z_1^2} = s_1 H$$

where $\beta' = \gamma\beta$. Likewise equations (A4) simplify to

$$\left.\begin{aligned}\gamma + z_1\beta' + 2Hs_1z_1\alpha &= s_1H\left(1+z_1^2\right)\\ H\gamma + Hz_2\beta' + 2s_2z_2\alpha &= s_2\left(1+z_2^2\right)\\ \gamma + z_3\beta' + 2Hs_3z_3\alpha &= s_3H\left(1+z_3^2\right)\\ H\gamma + Hz_4\beta' + 2s_3z_4\alpha &= s_4\left(1+z_4^2\right)\end{aligned}\right\} \qquad \text{(A6)}$$

If we call D the determinant of the coefficients, equations (A6) have the solution

$$\left.\begin{aligned}
\beta' D &= H\left\{s_2 s_3\left[z_3\left(1+z_2^2\right)-z_2\left(1+z_3^2\right)\right]-H^2 s_1 s_3\left[z_3\left(1+z_1^2\right)-z_1\left(1+z_3^2\right)\right]\right. \\
&\qquad \left. +\, s_1 s_2\left[z_2\left(1+z_1^2\right)-z_1\left(1+z_2^2\right)\right]\right\} \\
\gamma D &= -H\left\{z_1 s_1 s_3\left[z_3\left(1+z_2^2\right)-z_2\left(1+z_3^2\right)\right]-H^2 z_2 s_1 s_3\left[z_3\left(1+z_1^2\right)\right.\right. \\
&\qquad \left.\left. -\, z_1\left(1+z_3^2\right)\right]+z_3 s_1 s_2\left[z_2\left(1+z_1^2\right)-z_1\left(1+z_2^2\right)\right]\right\} \\
\alpha D &= \frac{1}{2} H^2\left[s_1\left(1+z_1^2\right)\left(z_3-z_2\right)+s_3\left(1+z_3^2\right)\left(z_2-z_1\right)\right]-\frac{1}{2} s_2\left(1+z_2^2\right)\left(z_3-z_1\right)
\end{aligned}\right\} \quad \text{(A7)}$$

and D is

$$D = H^2\left[s_1 z_1\left(z_3-z_2\right)+s_3 z_3\left(z_2-z_1\right)\right]-s_2 z_2\left(z_3-z_1\right)$$

These quantities can be substituted back into equations (A6) to find H. The result is the quadratic equation for H^2

$$\begin{aligned}
&H^4 s_1 s_3\left(z_4-z_2\right)\left(z_3-z_1\right)\left(1-z_1 z_3\right)-H^2\left\{\left(z_4-z_3\right)\left(z_2-z_1\right)\left[s_3 s_4\left(1-z_3 z_4\right)\right.\right. \\
&\qquad \left.\left. +\, s_1 s_2\left(1-z_1 z_2\right)\right]+\left(z_4-z_1\right)\left(z_3-z_2\right)\left[s_2 s_3\left(1-z_2 z_3\right)+s_1 s_4\left(1-z_1 z_4\right)\right]\right\} \\
&\qquad +\, s_1 s_4\left(z_4-z_2\right)\left(z_3-z_1\right)\left(1-z_2 z_4\right)=0
\end{aligned}$$

Of the two values of H^2 that result, the positive one is chosen. If they are both positive, the one that yields the smaller value of h is used. Once H has been found, equations (A7) give the parameters α, β, and γ for the four points chosen. The process continues as explained until all the data points are encompassed within the equal-error curve. Then all the parameters have been found for the curve that fits the data with the smallest maximum error, and the value of h found in the process gives a measure of how well the data have been fitted.

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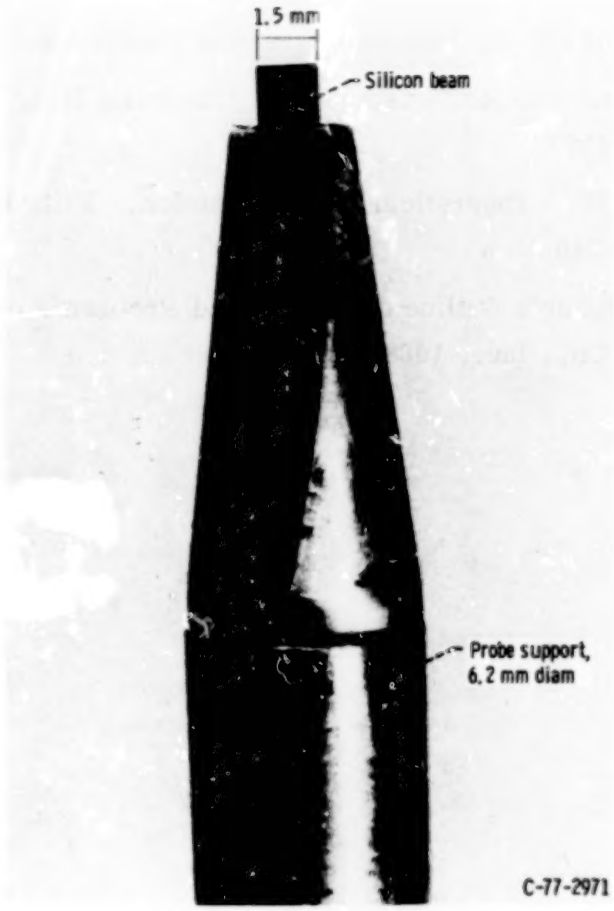

Figure 1. - Drag-force anemometer.

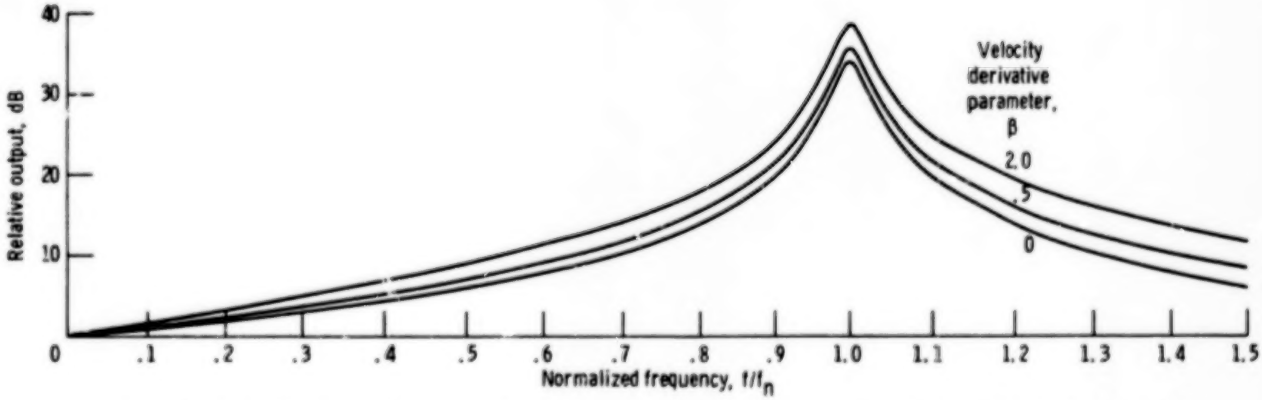

Figure 2. - Theoretical frequency response for various values of velocity derivative parameter. Damping coefficient, ζ, 0.01.

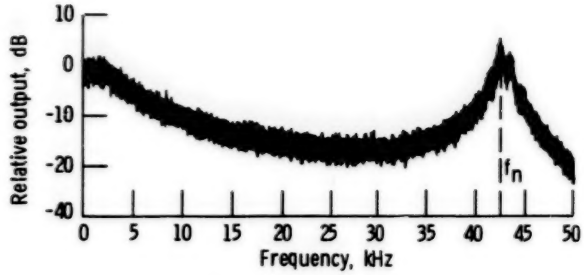

Figure 3. - Output signal spectrum of typical drag-force anemometer in open jet.

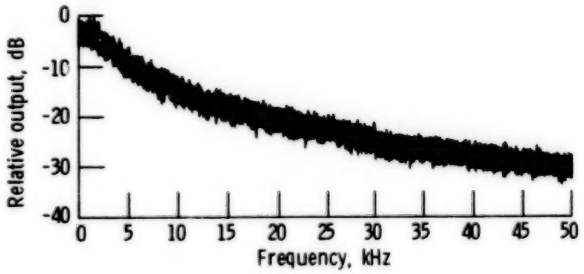

Figure 4. - Output signal spectrum of hot-wire anemometer in open jet.

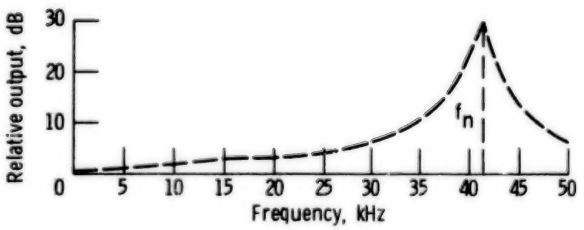

Figure 5. - Frequency response of drag-force anemometer.

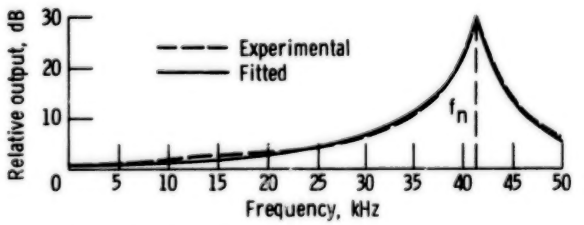

Figure 6. - Experimental and fitted frequency response curves.

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15. Supplementary Notes

16. Abstract

This report describes an experiment designed to determine the dynamic behavior of a drag-force anemometer in high-frequency, unsteady flow. In steady flow the output of the anemometer is proportional to stream velocity head and flow angle. Fluid mechanics suggests that, in unsteady flow, the output would also be proportional to the rate of change of fluid velocity. It was determined from the experiment described herein that effects due to the rate of change of fluid velocity are negligible for the probe geometry and frequencies involved.

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